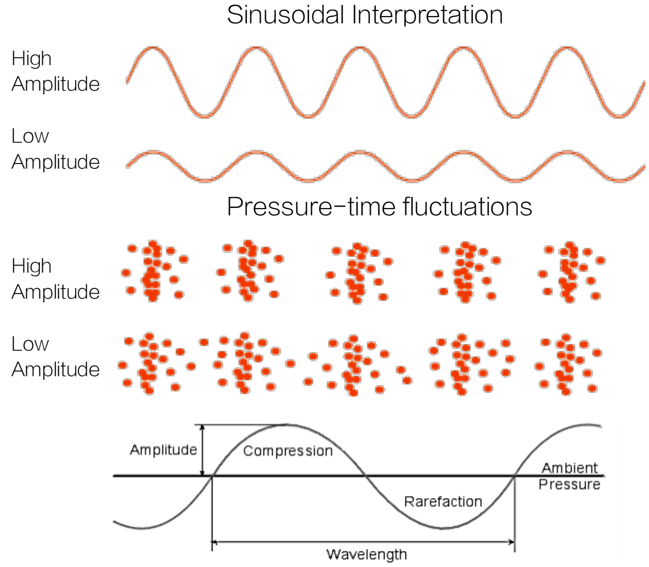
THE PHYSICS OF SOUND AND MUSIC

A Sound Wave is characterized as a mechanical, longitudinal and pressure wave. It is a pressure wave that consists of compressions (high pressure) and rarefactions (low pressure) of air particles. Unlike light, sound is not on the electromagnetic spectrum and needs a medium (e.g. air) to travel through.



Sources of Sound

The source of any sound is a vibrating object, at a frequency usually too high to be visibly detected, but it can be detected by our ears. Vibrating strings and air columns.

Speed of Sound= **343m/s** in air
v solids > v liquids > v gases

$x = vt$

$v = f\lambda$

The acoustic wave equation describes sound waves in a liquid or gas.

$$\frac{\partial^2 X}{\partial t^2} = k\frac{\partial^2 X}{\partial x^2} \quad = \quad \frac{\partial^2 X}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 X}{\partial x^2} \quad = \quad p = p_0 \sin(2\pi f t \mp kx)$$

Sine wave equation:

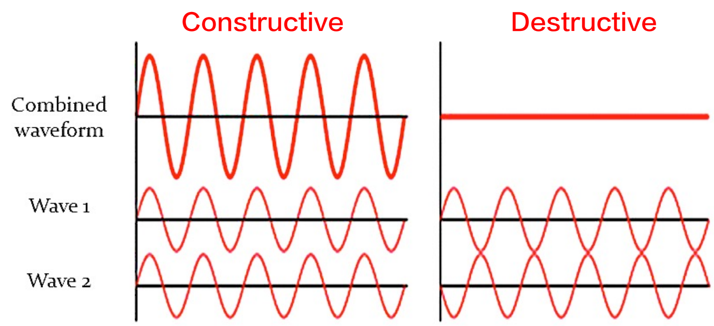
$$y(t)=A\sin(kx-2\pi ft+\varphi) \qquad k= 2\pi f / v = 2\pi/\lambda$$

$$y(t)=A\sin(\omega t)$$

CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

If repeated **constructive interference** of two compressions (anti-nodes) is followed by repeated constructive interference of two rarefactions, a very LOUD sound is produced because of the intense oscillation of the particles from high to low pressure.

Destructive interference (180° out of phase, node) results in no displacement of the particles, important concept for concert halls and noise cancelation headphones.



SOME COMMON SOUNDS AND THEIR INTENSITIES

Source	Intensity	x TOH
Threshold of Hearing (TOH)	**0 dB**	10^{0}
Rustling Leaves	10 dB	10^{1}
Whisper	20 dB	10^{2}
Normal Conversation	**60 dB**	10^{6}
Busy Street Traffic	70 dB	10^{7}
Vacuum Cleaner	80 dB	10^{8}
Large Orchestra	98 dB	$10^{9.8}$
Walkman at Maximum Level	100 dB	10^{10}
Front Rows of Rock Concert	110 dB	10^{11}
Threshold of Pain	130 dB	10^{13}
Military Jet Takeoff	140 dB	10^{14}
Instant Perforation of Eardrum	**160 dB**	10^{16}

Quality of Sound

Waveform= Timbre
Time interval= Duration
Amplitude = Loudness
Frequency= Pitch Frequency 1 Hz= 1 vibration/sec

Frequency measures the cycle rate of the physical waveform, pitch is how high or low it sounds to the ear.

Why decibels?

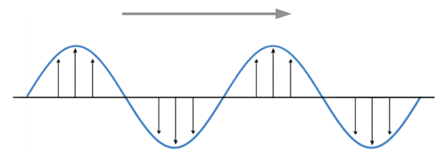
Ears judge loudness on a logarithmic scale

$$\text{Loudness } \beta = I_{dB} = 10\log_{10}\left(\frac{I}{I_0}\right)$$

The human range of hearing is **20 to 20,000 Hz**. The upper limit decreases with age.

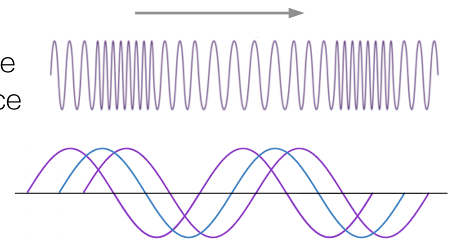
TYPES OF SOUND WAVES

Transverse Wave



For transverse waves the displacement of the medium is perpendicular to the direction of propagation of the wave. They cannot propagate in a gas or a liquid because there is no mechanism for driving motion perpendicular to the propagation of the wave. Strings and string instruments encompass this type of wave.

Longitudinal Traveling Wave



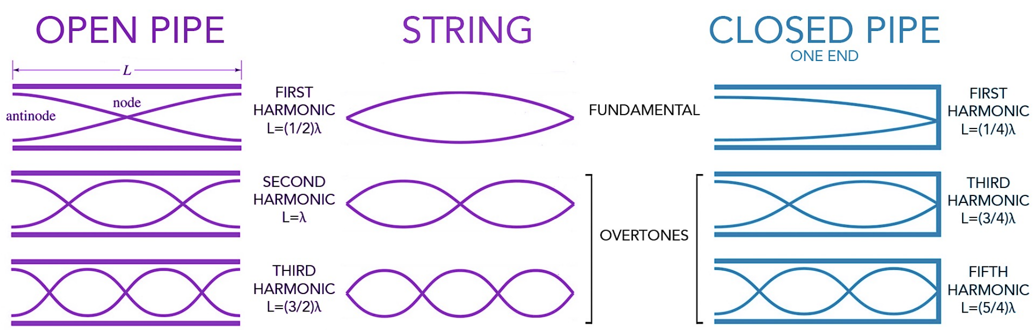
In longitudinal waves the displacement of the medium is parallel to the propagation of the wave. Traveling waves are not confined to a given space and move parallel along the medium. Vibrations in air are called traveling longitudinal waves, which we can hear.

Longitudinal Standing Wave

Standing waves are confined to a given space, longitudinal and stationary. When the proper frequency is used, the interference of the incident wave and the reflected wave occur such that there are specific points that appear to be standing still. These waves are formed from vibrations in a tube (wind instruments). Only the waves that can fit in the tube will resonate, other frequencies are lost. The longest wave that can fit in the tube is the **fundamental** and the others are **overtones.**

Harmonics

A harmonic is a wave with a frequency that is a positive integer multiple of the original or fundamental wave.

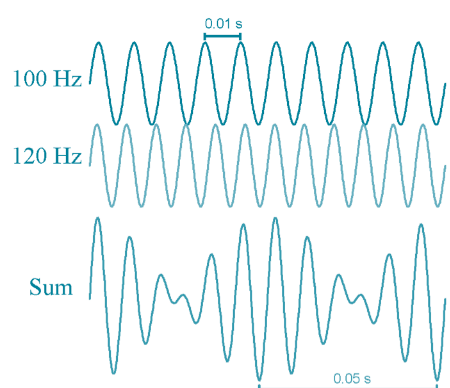


$L = \frac{n}{2}\lambda \quad or \quad L = \frac{n_{odd}}{4}\lambda$

Diagram shows particle displacement
Note that these depictions of open and closed pipes are sinusoidal representations of pressure-time fluctuations. Particle displacement and pressure are inversely proportional.

BEATS-INTERFERENCE IN TIME

A beat is an interference pattern between 2 or more sounds with slightly different frequencies(< ~20 or 30 Hz). It is perceived as a periodic variation in volume with a rate or frequency at the difference frequency. If the frequency difference is larger than ~ 20 or 30 Hz, a dissonant tone is usually perceived rather than distinct beats. In addition, as the two tones gradually approach unison, the beating slows down to a point that is imperceptible. For complex sounds, beats can arise from any of the partials of the sounds.



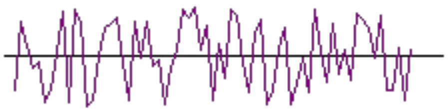
$$\sin(w_1 t) + \sin(w_2 t) = 2\sin(w_3 t)\cos(w_4 t)$$

w_3 = average of w_1 and w_2 ; w_4 = 1/2 of their difference.
Ex. 440 Hz and 450 Hz, you would hear 445 Hz beating at 10 times per second. When comparing two notes, the ear most easily picks up "beats.".

Two important concepts in music and sound are consonance and dissonance. Too much **consonance** in music makes it easy to listen to, but a little bland. **Dissonance** adds a powerful tension, but too much can make music hard to connect with.

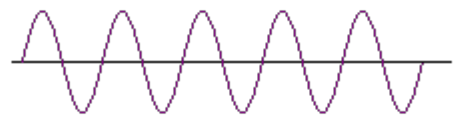
Noise

Noise does not have a readily discernible relationship. Its waveform is jerky and irregular and there is no strong regularity for your ear to pick up a musical tone. Dissonant sounds are displeasing and have an irregular and non-repeating pattern.

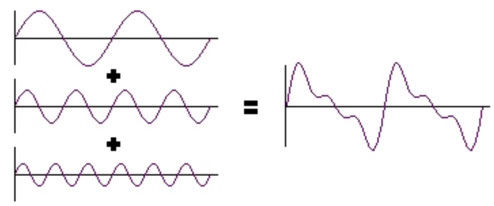


Music

Music is a mixture of sound waves, typically with whole number ratios between frequencies associated with the notes, a discernible mathematical relationship. The waveform is a regular series of repeated cycles.



Natural Frequency



Natural frequency is the frequency at which a system oscillates when not subjected to a continuous or repeated external force. Any object that vibrates will create a sound able to be detected by human if it has a big enough amplitude. They tend to vibrate at a particular frequency or a set of frequencies. If the object vibrates at a single frequency it will produce a **pure tone**, sine waves that have no overtones, like the flute. Middle C would be a sound wave of 256 Hz. Musical objects, like a tuba, will vibrate at a set of frequencies that are mathematically related by whole number ratios. **Resonance** occurs whenever a physical system is driven at its natural frequency. An objects set of natural frequencies and its resonance is visualized in the **Chladni** plates below. Lines represent the nodal positions.

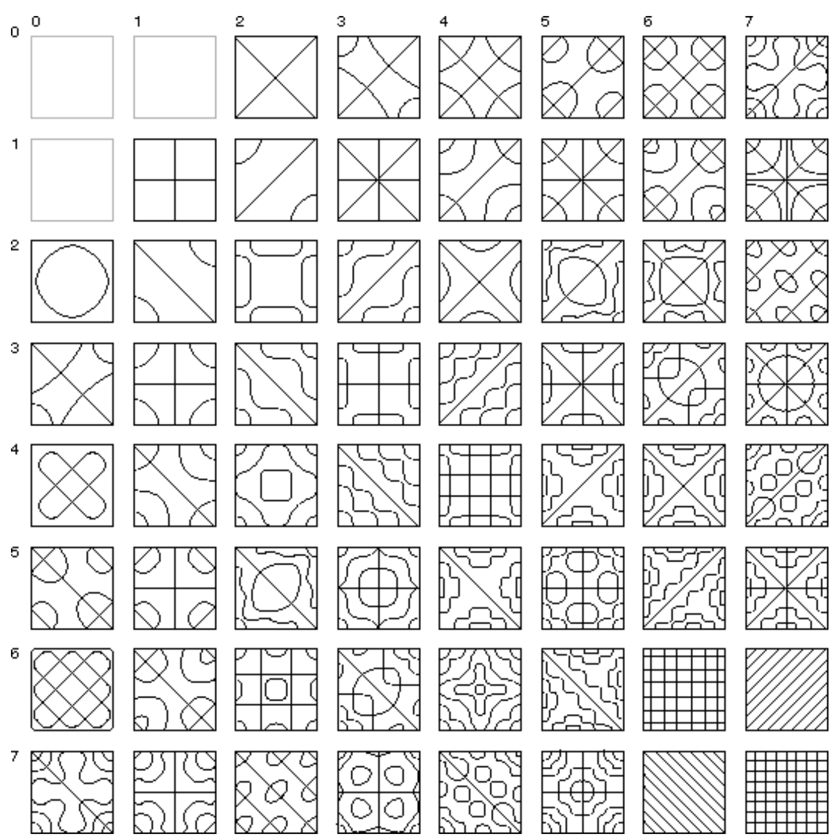
Factors affecting natural frequency:

String: linear density, tension, length; Tube: room temperature, length. Ex. Tuning a guitar simply means setting the fundamental frequency of each string to the correct value. (↑ tension = ↑ fundamental frequency)

Overtones

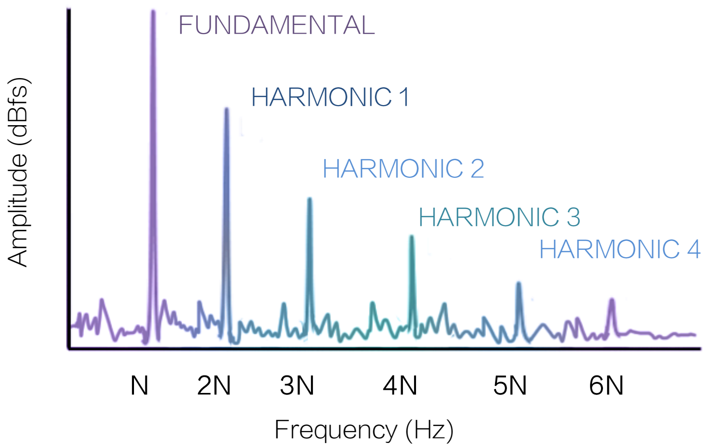
Overtones are multiples of the fundamental. The quality (or timbre) of a sound depends on the presence of overtones, specifically their number and their relative amplitudes. Generally, when a note is played on a musical instrument, the fundamental and overtones are present simultaneously. Instruments of different shapes and actions produce different overtones. The overtones combine to form the characteristic sound of the instrument. You could say that the sound spectrum of an instrument is its "acoustical fingerprint. Notes played on different instruments will have a fundamental and overtones of different types and powers/dominance.

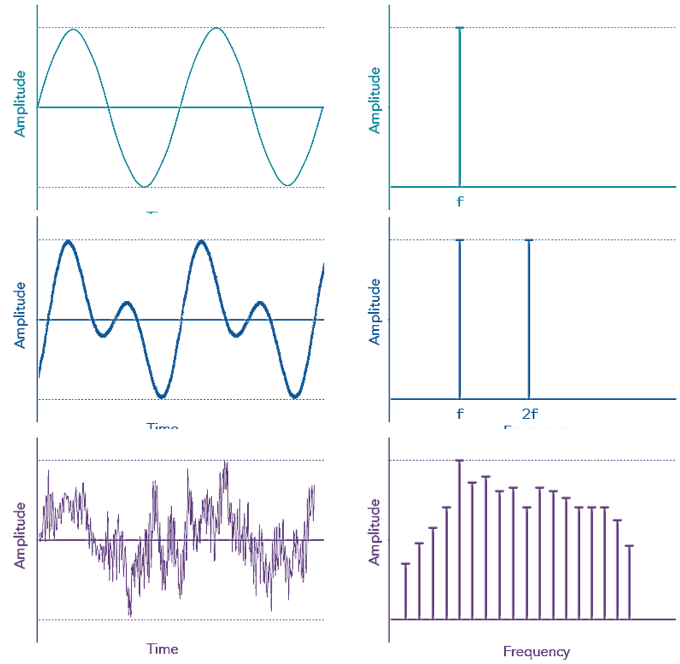
Chladni Plates, square, organized by axis

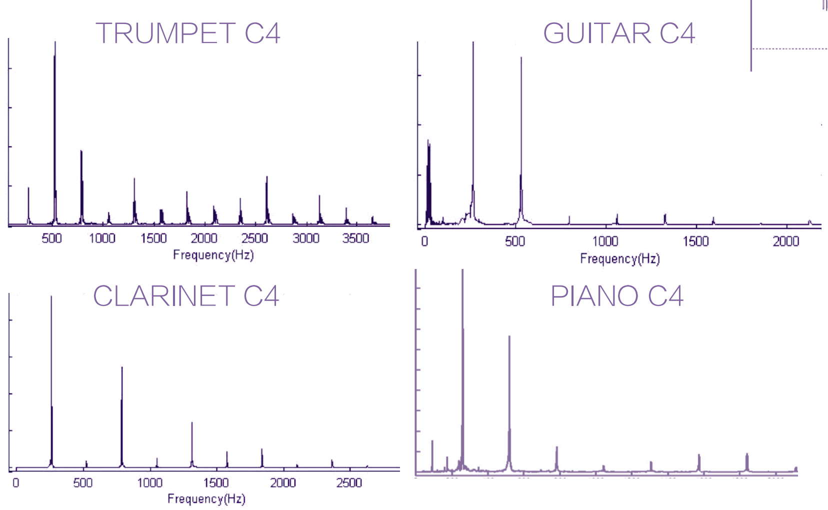


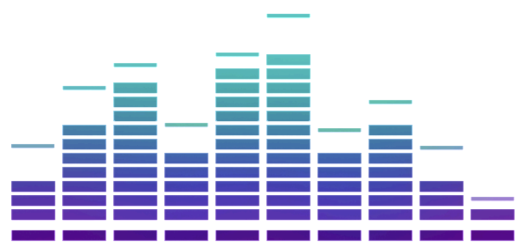
FOURIER ANALESES

Fourier analysis is a mathematical technique that decomposes sound into its component frequencies. Each instrument would have a unique spectra of harmonics.

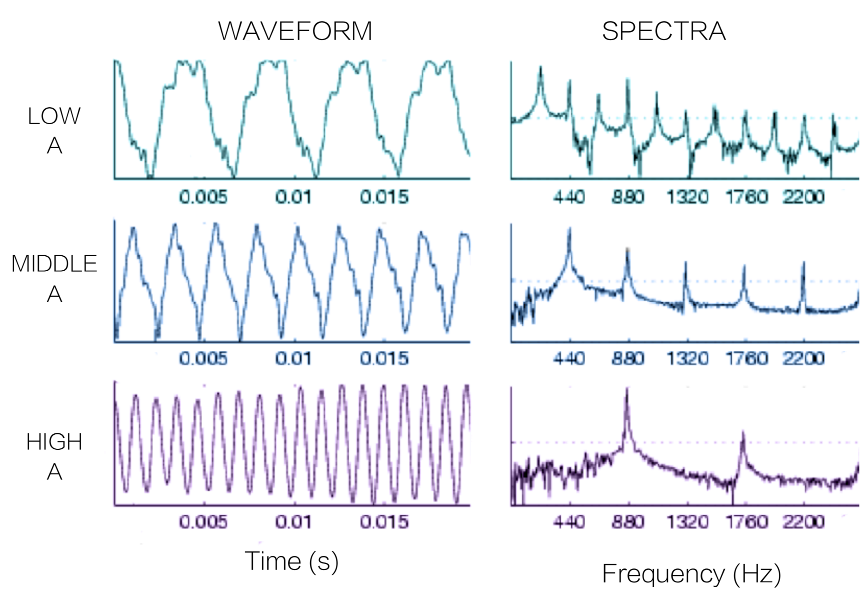








Fourier analysis provides mathematical evidence of the different timbers of instruments. Any periodic wave (any wave that consists of a consistent, repeating pattern) can be broken down into simpler waves. That is, it can be written as the sum of a bunch of simpler waves. Mathematicians often use sine and cosine waves as the simple waves because of their predefined and simple nature.



MUSICAL SCALES

There have been many scales or tuning methods that have been developed over time, from simple to complex. However, none of them are perfect for all music because music is based on intervals between notes. The intervals don't always match up when the key is changed. The main scales include the Pythagorean Scale, the Scale of Just Intonation, and the Equal Temperament Scale. See the ratios to fundamental below.

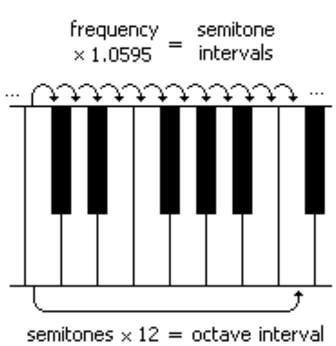
Note	Pythagorean Scale	Just Scale	Equal Tempered Scale
C	1.000	1.000	1.000
D	9/8=1.125	9/8=1.125	$2^{2/12}$=1.12246
E	81/64=1.2656	5/4=1.2500	$2^{4/12}$=1.25992
F	4/3=1.3333	4/3=1.3333	$2^{5/12}$=1.33483
G	3/2=1.500	3/2=1.500	$2^{7/12}$=1.49831
A	27/16=1.6875	5/3=1.6667	$2^{9/12}$=1.68179
B	243/128=1.8984	15/8=1.875	$2^{11/12}$=1.88775
C	2.000	2.000	2.000

A problem with the Pythagorean and Just scales is that songs can not be easily transposed to another key. The Equal Temperament scale attempts to correct the frequency spacing problem without losing the special intervals. So, instead of working with intervals that relate to each other in mathematical harmony, equal temperament divides the octave into 12 equal pieces. However, some tones are more than a 4 Hz difference than their Just counterparts. This is a clear audible difference so instead of tones strengthening each other, they destructively interfere with each other.

The Chromatic Scale or Equal Tempered Scale

In western music, the set of all musical notes is called the **Chromatic Scale**

The twelfth root of two = 1.0595



The frequency of a note, when multiplied by this number gives the frequency of the next note up= **semitone**

The note A which is above Middle C has a frequency of **440 Hz**. It is often used as a reference frequency for tuning musical instruments.

FREQUENCY CHART

The following equations give the frequency of the nth key of a piano or the key number if starting with frequency.

$$f(n) = (\sqrt[12]{2})^{n-49} \times 440 \text{ Hz} \qquad f(n) = 2^{\frac{n-49}{12}} \times 440 \text{ Hz} \qquad n = 12log_2\left(\frac{4}{440\ Hz}\right) + 49$$

The following equation gives the absolute frequency, P_n, of a note in 12 tone equal temperament scale.

$$P_n = P_a(\sqrt[12]{2})^{n-a}$$

Where P_a refers to the frequency of a reference pitch (usually 440Hz). n refers to the numbers assigned to the desired pitch and and a to the number assigned to the reference pitch,

Table of Frequencies

Octave	C	C#	D	Eb	E	F	F#	G	G#	A	Bb	B
0	16.35	17.32	18.35	19.45	20.60	21.83	23.12	24.50	25.96	27.50	29.14	30.87
1	32.70	34.65	36.71	38.89	41.20	43.65	46.25	49.00	51.91	55.00	58.27	61.74
2	65.41	69.30	73.42	77.78	82.41	87.31	92.50	98.00	103.8	110.0	116.5	123.5
3	130.8	138.6	146.8	155.6	164.8	174.6	185.0	196.0	207.7	220.0	233.1	246.9
4	261.6	277.2	293.7	311.1	329.6	349.2	370.0	392.0	415.3	440.0	466.2	493.9
5	523.3	554.4	587.3	622.3	659.3	698.5	740.0	784.0	830.6	880.0	932.3	987.8
6	1047	1109	1175	1245	1319	1397	1480	1568	1661	1760	1865	1976
7	2093	2217	2349	2489	2637	2794	2960	3136	3322	3520	3729	3951
8	4186	4435	4699	4978	5274	5588	5920	6272	6645	7040	7459	7902

Middle C	**C4**
Standard tuning fork	**A4**
Piano range	**A0–C8**
Guitar strings	**E2, A2, D3, G3, B3, E4**
Bass strings	(5th) **B0**, (4th) **E1, A1, D2, G2**
Violin strings	**G3, D4, A4, E5**
Viola strings	**C3, G3, D4, A4**
Cello strings	**C2, G2, D3, A3**

Audible Frequency Range
20 Hz to 20,000 Hz
Wavelength range
3.74×10^{13} to 2.99×10^{16} nm

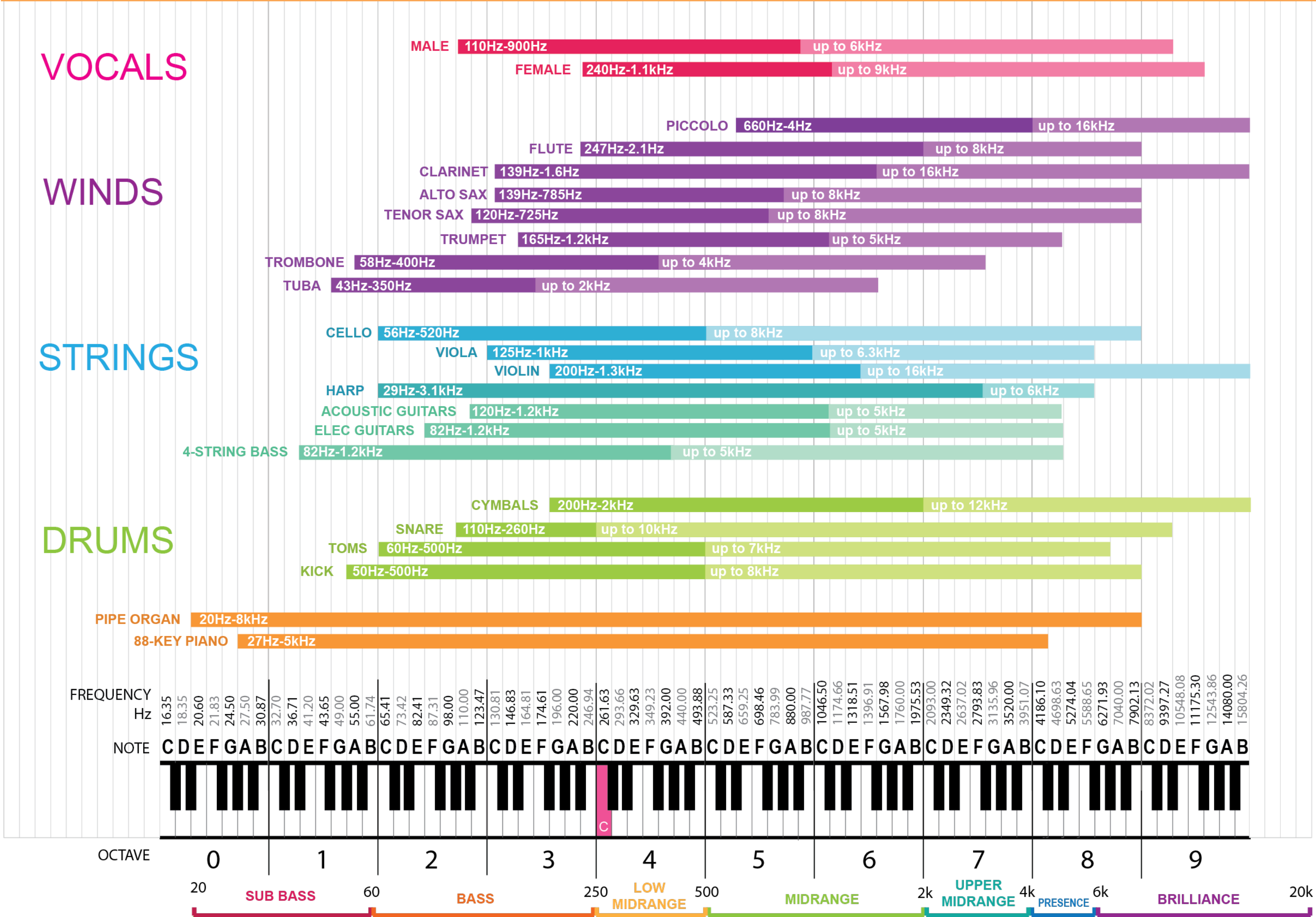
Visible Frequency Range
4.28×10^{14} to 7.69×10^{14} Hz
Wavelength Range
390 nm to 700 nm

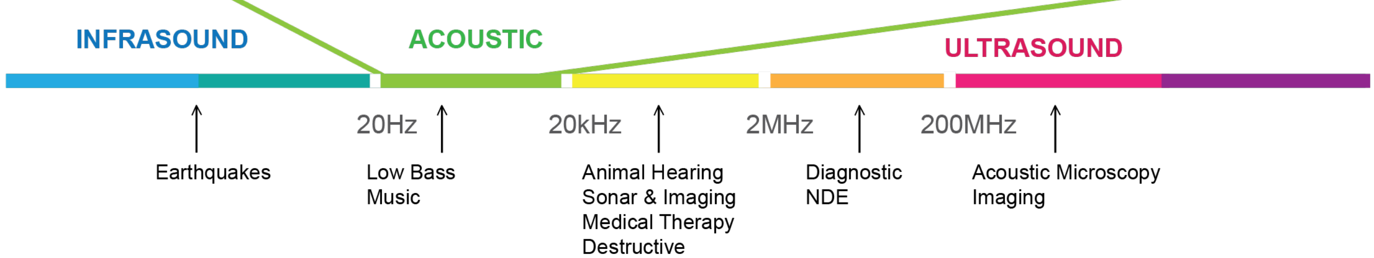
HARMONIES

An octave, a fifth, and a fourth are especially pleasing 2:1 3:2 4:3. The addition of two waveforms to a base note does not create peaks that are close together and hence creates pleasant sounds. The 'beating' is eliminated. So, for harmonies, you'd want to have a combination of sound waves that do not have peaks that create internal frequencies (spikes) that causes odd frequency sounds to occur.

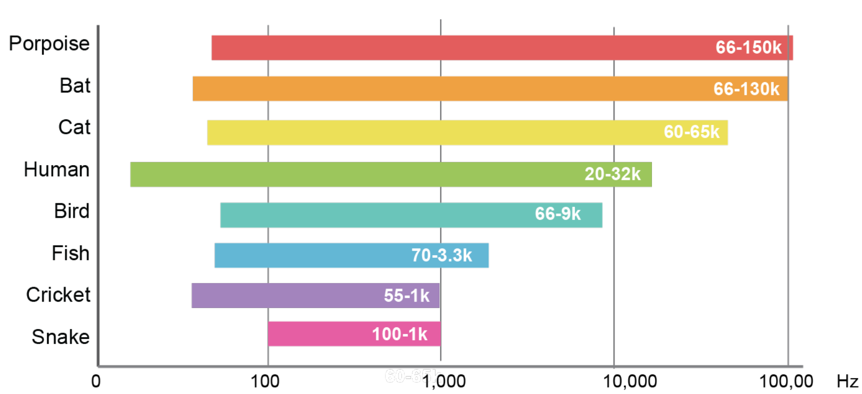
Chromatic Scale (C4–C5)		Interval size (from C)	Harmonic	Semitone	Ratio	Notes
C	261.63 Hz	**Perfect unison**	1, 2, 4, 8,16	0	1:1	Strongly(perfect) consonant, identical
C♯ / D♭	277.18 HZ	**Minor second**	17	1	16:15	Strongly dissonant
D	293.66 Hz	**Major second**	9, 18	2	9:8	Less dissonant
D♯ / E♭	311.13 Hz	**Minor third**	19	3	6:5	Imperfect consonant, basis of minor chords and scales, melancholy flavor
E	329.63 Hz	**Major third**	5, 10, 20	4	5:4	Imperfect consonant, stable sound, basis of major chords and scales
F	349.23 Hz	**Perfect fourth**	21	5	4:3	Mildly dissonant, a stretched feeling, can be consonant or dissonant, when not supported by a lower third or fifth (but see below).
F♯ / G♭	369.99 Hz	**Tritone**	11, 22, 23	6	25:18	Dissonant, often found in chords with 4 notes, known as the Devil's interval. Rarely used, banned in the renaissance church music
G	392.00 Hz	**Perfect fifth**	3, 6, 12 ,24	7	3:2	Strongly(perfect) consonant, found in both minor and major chords. Add solidness but not much character to harmony
G♯ / A♭	415.30 Hz	**Minor sixth**	13, 25, 26	8	8:5	Imperfect consonant
A	440.00 Hz	**Major sixth**	27	9	5:3	Imperfect consonant
A♯ / B♭	466.16 HZ	**Minor seventh**	7, 14, 28, 29	10	9:5	Mildly dissonant
C	523.25 Hz	**Perfect octave**	1, 2, 4, 8,16	12	2:1	Strongly consonant, like unison

SONIC FREQUENCY CHART





COMMON ANIMALS AND THEIR FREQUENCY RANGES

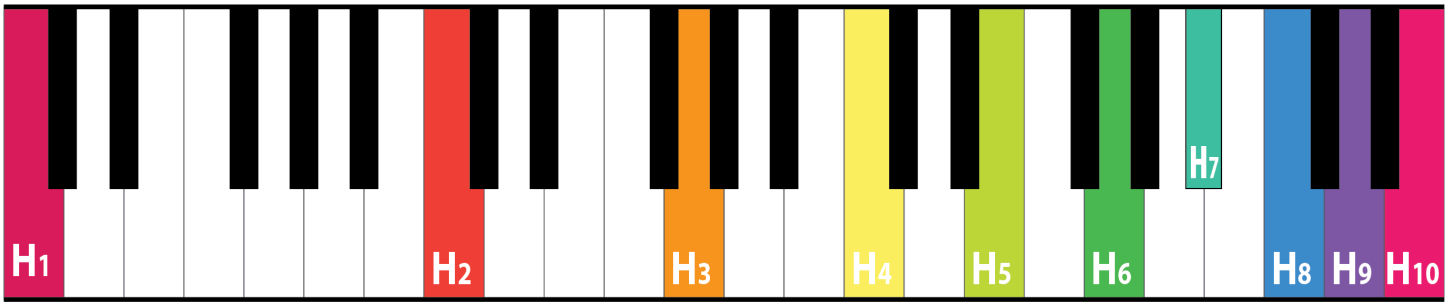


The Just Noticeable Difference (JND)

1 Hz up to 1,000 Hz
2 Hz at 2,000 Hz
10 Hz at 4,000 Hz

Compared to the eye, from red to violet, the frequency of light almost doubles. This means that the eye is only sensitive over about one octave. The ear, as a receptor, has a large frequency range and can accommodate sounds with vastly different loudness.

HARMONICS



Interval										
	Octave	Perfect 4th	Major 3rd	Minor 2nd	Major 2nd	Minor 2nd	Minor 2nd	Minor 2nd	Minor 2nd	Minor 2nd
	Perfect 5th	Major 3rd	Minor 3rd	Major 2nd	Major 2nd	Major 2nd	Minor 2nd	Minor 2nd	Minor 2nd	

Harmonic	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	C	C	G	C	E	G	A♯	C	D	E	F♯	G	A	A♯	B	C	C♯	D	D♯	E
C♯ / D♭	C♯	C♯	G♯	C♯	F	G♯	B	C♯	D♯	F	G	G♯	A♯	B	C	C♯	D	D♯	E	F
D	D	D	A	D	F♯	A	C	D	E	F♯	G♯	A	B	C	C♯	D	D♯	E	F	F♯
D♯ / E♭	D♯	D♯	A♯	D♯	G	A♯	C♯	D♯	F	G	A	A♯	C	C♯	D	D♯	E	F	F♯	G
E	E	E	B	E	G♯	B	D	E	F♯	G♯	A♯	B	C♯	D	D♯	E	F	F♯	G	G♯
F	F	F	C	F	A	C	D♯	F	G	A	B	C	D	D♯	E	F	F♯	G	G♯	A
F♯ / G♭	F♯	F♯	C♯	F♯	A♯	C♯	E	F♯	G♯	A♯	C	C♯	D♯	E	F	F♯	G	G♯	A	A♯
G	G	G	D	G	B	D	F	G	A	B	C♯	D	E	F	F♯	G	G♯	A	A♯	B
G♯ / A♭	G♯	G♯	D♯	G♯	C	D♯	F♯	G♯	A♯	C	D	D♯	F	F♯	G	G♯	A	A♯	B	C
A	A	A	E	A	C♯	E	G	A	B	C♯	D♯	E	F♯	G	G♯	A	A♯	B	C	C♯
A♯ / B♭	A♯	A♯	F	A♯	D	F	G♯	A♯	C	D	E	F	G	G♯	A	A♯	B	C	C♯	D
B	B	B	F♯	B	D♯	F♯	A	B	C♯	D♯	F	F♯	G♯	A	A♯	B	C	C♯	D	D♯